

## REVIEWS

*Edited by* CATHERINE GOLDSTEIN AND PAUL R. WOLFSON

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**Lakatos' Philosophy of Mathematics: A Historical Approach.** By Teun Koetsier. Studies in the History and Philosophy of Mathematics. Volume 3. Amsterdam (North-Holland, Elsevier). 1991. 312 pages, including bibliography and index.

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Imre Lakatos' sudden death in 1974 prematurely ended the philosophical contributions of one of the most underappreciated, yet widely influential, philosophers of the 20th century. Many scholars remember his challenging contribution found in the Schilpp two-volume *Library of Living Philosophers* series on Popper (Open Court, 1974), whom Lakatos evidently admired enormously. Others may know him from the posthumous *Proofs and Refutations: The Logic of Mathematical Discovery*, excellently edited by Worrall and Zahar (Cambridge Univ. Press, 1976). Those with a more determined interest may also be familiar with the two-volume set of Lakatos' papers, edited by Worrall and Currie, published as *The Methodology of Scientific Research Programs and Mathematics, Science and Epistemology* (Cambridge Univ. Press, 1978).

Teun Koetsier's *Lakatos' Philosophy of Mathematics* is a worthy addition to the Lakatos literature, an exploration of "quasi-empiricism" and other issues in the methodology of mathematics. Lakatos courageously explored the nature of proof

theory in mathematics, a less than popular inquiry in many quarters. This required a careful examination of mathematical theory, methodology, confirmation, disconfirmation, and universalization of mathematical results. Lakatos found fault with those who believe that the foundations of mathematics transcend demonstration—correctly elucidating that the beginning point in mathematics, as in science, must begin with a leap of faith. But Lakatos was also correct to argue that not all leaps of faith are equally barren of justification, and that some first principles or starting points are more justified than others, even if no level of justification reaches certainty. An important part of this argument is Lakatos' claim that mathematics is not Euclidean, with truth flowing downward from the axioms at the top to the proofs underneath, thereby inundating the entire system. Rather, for Lakatos, mathematics is "quasi-empirical," using theories characterized by the upward transmission of falsity from "basic statements" to the axioms.

Such a realistic, a posteriori assessment of mathematics provides a profound shift in methodology:

The development of Euclidean theory consists of three stages: first the naive pre-scientific stage of trial and error which constitutes the prehistory of the subject; this is followed by the foundational period which reorganizes the discipline, trims the obscure borders, establishes the deductive structure of the safe kernel; all that is then left is the solution of problems inside the system, mainly proofs or disproofs of interesting conjectures. . . .

The development of a quasi-empirical theory is very different. It starts with problems followed by daring solutions, then by severe tests, refutations. The vehicle of progress is bold speculations, criticism, controversy between rival theories, problemshifts. Attention is always focussed on the obscure borders. The slogans are growth and permanent revolution, not foundations and accumulation of eternal truths.

The main pattern of Euclidean criticism is suspicion: Do the proofs really prove? Are the methods used too strong and therefore fallible? The main pattern of quasi-empirical criticism is proliferation of theories and refutation. [Lakatos, *Philosophical Papers*. Vol. 2. pp. 29–30, cited by Koetsier on pp. 60–61].

These strident views demand development. The great merit of Koetsier's work is that it illuminates and builds upon the bountiful intellectual legacy left by Lakatos, offering a discussion in "the methodology of mathematics" (p. 1). Over 10 chapters, Koetsier examines methodological and philosophical problems that equal "an essay in quasi-empiricism in every sense of the word" (p. 3).

Koetsier begins in chapter one by examining Lakatos' "fallibility thesis"—that fallibility is an essential characteristic of mathematics. Following Lakatos, Koetsier examines and rejects a strong fallibility thesis—that in principle a whole theory can be refuted. If this position were accepted, it would imply that the continuity of mathematics and the accumulation of truth would be accidental, at best. But there is an important, strong continuity to mathematics—a history that is interwoven and progressive. So Lakatos and Koetsier look elsewhere, uncovering important "heuristic approaches" or "patterns" in the historical development of mathematics that indicate a "weak fallibility" in mathematical knowledge—in other words, that mathematical theories are fallible, yet not refutable in their totality. This makes the continuity of mathematics explainable, built on theories that are never totally wrong, but stand in need of refinement and shaping.

The second chapter is a look at falsification in mathematics, a search for potential falsifiers through the use of Lakatos' methodology of scientific research programmes (here the British spelling is used in the text). These falsifiers could only be products of the underlying mathematical theories used in constructing the paradigm—the research programmes or developing theories at play. Koetsier concludes that the refutation of a research programme is not a logical refutation, but a statement that another research programme is more competitive and can replace the original programme.

While mathematics may be fallible, this does not suggest a purely scientific model. Mathematics does have its own contextual rationality. Chapters III–V are perhaps best described as “case studies” in an evaluation of mathematical rationality in terms of a methodology of scientific research programmes, touching on the work of Cauchy, Spalt, and Giorello, in that order. In each case we see evidence of a scientific programme, yet instructive differences that underline the unique programmatic development of mathematics.

These case studies help to showcase mathematical methodology. Chapter VI follows with an attempt to define a “methodology of mathematical research traditions” (p. 151), characterized by historically identifiable common general assumptions about the mathematical entities studied within a domain, and the assumptions about how to prove properties of those entities. By looking at various traditions in mathematics, Koetsier speculates about the proof theory relevant to each tradition, and the factors influencing the weight of conjectures and theorems (see especially pp. 170–1).

Of compelling interest are the last four chapters. Chapter VII presents evidence to indicate a pre-Euclidean “Demonstrative Tradition” in mathematics, which would ground the subject in what we would now recognize as a more scientific methodology. Chapter VIII then jumps to the 18th century, outlining a “turn” from a formalist, nondeductive system to a conceptual, deductive approach in the 19th century. Chapter IX offers another case study, this time a look at the interchangeability theorem for partial differentiation, showing at least one example of how mathematical research traditions change over time. Finally, in the last chapter, Koetsier presents his conclusions, that mathematics is fallible, that this fallibility is “weak,” and that aligning mathematics with a realist position “most naturally explains the development of mathematics” (p. 7).

By way of evaluation, Koetsier undertakes an ambitious project. I am not sure that many mathematicians will feel comfortable with his suggestion that mathematical theories and methodologies are not *a priori*, and that mathematics has much in common with science. Understanding this, some may argue that Koetsier has not found enough similarity for concern, and in particular that his flirtation with Popper and Kuhn involves a dangerous misunderstanding of mathematics, science and the evolution of paradigms. But following Lakatos' lead, Koetsier attempts to prove that the imposing edifice of mathematics has less than solid structural support. It is no good arguing against either Lakatos or Koetsier by relying on a traditional, deductive picture of mathematics—that is the view challenged and the ground upon

which they wish to conduct this debate. Perhaps critics could argue that there is a difference between mathematical truth and the understanding of mathematics—that it is only our understanding of mathematical truths that is in need of refinement, not that mathematics is “quasi-empirical.” Surprisingly, the best refutation of Lakatos and Koetsier may be from inside science itself, showing that Popper is shockingly mistaken, that science is really “risk analysis,” that theories are never completely falsified, or that some inductive evidence equals virtual certainty—traditional counterresponses designed to threaten Popper which may show that the falsifiability approach ultimately has little to do with mathematics. A traditional, realistic approach to science may also damage the reliance by Lakatos and Koetsier on Kuhn and paradigm shifts.

This is a well-written, clear, and easily comprehensible text, full of internal summaries and quite devoid of rigid technical language. While not a book for beginners, it has the singular advantages of simplicity and clarity, both highly relevant in producing good argumentation. But two picky notes—first, there are a few misspellings that have escaped the editor and the author, a problem all too common in works targeted primarily at professional audiences, and, second, the text follows *Webster's* preferred “an historical” (for example, see p. 15). This makes the less preferable use of “a historical” in the title odd and inconsistent. Overall, I found the work to be thoughtful, very convincing, and a pleasure to read. I highly recommend this important book to anyone interested in the philosophy of mathematics, philosophical logic, and the history of mathematics. Koetsier is to be congratulated on a landmark contribution to the strangely sparse discussion of Lakatos' work and influence.

**Berkeley's Philosophy of Mathematics.** By Douglas M. Jesseph. Chicago and London (The University of Chicago Press). 1993. xii + 322 pp.

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Jesseph's book naturally divides in two parts. The first is devoted to Berkeley's conception of geometry and arithmetic; its key is his rejection of abstract ideas and the theory that mathematical objects are abstracted from experience. It begins with a careful consideration of Berkeley's position against its background—in an extended sense, which goes from the role of abstraction in the philosophy of mathematics of the Aristotelian Scholastics, through the conceptions of such 17th-century mathematicians as Barrow and Wallis, to the rejection of abstraction in the thought of Peter Browne, a teacher at Trinity College, Dublin, in the years when Berkeley was a student there. Having thus set the stage, the author proceeds to show the development of Berkeley's philosophy of mathematics, from the initial stages, when he is concerned to refute abstractionism even at the price of rejecting classical geometry, to the reinterpretation of this latter in the *Principles*, to the final full